symmetries. We mentioned above that rotational symmetries can introduce topological states in crystalline systems; one may wonder what topological states appear in QCs because of their 'forbidden' symmetries.

Apart from a topological characterization that relies on the QP order itself, QP systems can serve as a convenient playground for testing other topological phases. Usually, when a topological state of matter is predicted, it is tested on periodic models, which may be modified with the addition of disorder. However, when using quasicrystalline systems, one samples a completely different regime of models, as they cannot simply be deformed into crystals[37]. For example, it was recently shown that a QC can be in a unique topological state, called a weak topological insulator, although this state apparently exists only in periodic structures[38].

Additionally, QP systems are used to study the interplay between disorder and topological properties, as they are intermediate between periodic and disordered systems. For example, the AAH model has a phase transition between extended and localized states as a function of the potential strength. Consequently, introducing a QP potential is a common method for mimicking disorder in 1D optical lattices. In such systems, special interest is devoted to the effect of quasiperiodicity on possible realizations of 1D topological superconductors. This celebrated state of matter hosts exotic fermion excitations at its edges and was recently observed in semiconducting wires[39–41]. Further studies show that these excitations persist also under the addition of QP potentials, which in turn may enrich the predicted topological properties[42–46] (see also the Commentary by Beenakker and Kouwenhoven[47]).

To summarize, QP systems are fascinating structures that intrigue physicists with their rich phenomena and elusive order. Their conjunction with topology opens a porthole to a new and exciting field of research where physics journeys between dimensions. ❐

Yaacov E. Kraus is in the Department of Physics, Holon Institute of Technology, Holon 5810201, Israel. Oded Zilberberg is in the Institute for Theoretical Physics, ETH Zürich, 8093 Zürich, Switzerland. e-mail: odedz@phys.ethz.ch

References

1. Janot, C. *Quasicrystals, a Primer* (Oxford Univ. Press, 1994).
2. Senechal, M. *Quasicrystals and Geometry* (Cambridge Univ. Press, 1996).
3. Shechtman, D., Blech, I., Gratias, D. & Cahn, J. W. *Phys. Rev. Lett.* **53,** 1951–1953 (1984).
4. Bindi, L., Steinhardt, P. J., Yao, N. & Lu, P. J. *Science* **24,** 1306–1309 (2009).
5. Macia, E. *Rep. Prog. Phys.* **69,** 397 (2006).
6. Dubois, J. M *Chem. Soc. Rev.* **41,** 6760–6777 (2012).
7. Dal Negro, L. *et al. Phys. Rev. Lett.* **90,** 055501 (2003).
8. Man, W. *et al. Nature* **436,** 993–996 (2005).
9. Vardeny, Z. V., Nahata, A. & Agrawal, A. *Nature Photon.* **7,** 177–187 (2013).
10. Ryu, S. *et al. New J. Phys.* **12,** 065010 (2010).
11. Fu, L. *Phys. Rev. Lett.* **106,** 106802 (2011).
12. Slager, R. J. *et al. Nature Phys.* **9,** 98–102 (2013).
13. Asorey, M. *Nature Phys.* **12,** 616–618 (2016).
14. Bohr, H. *Acta Math.* **46,** 101–214 (1925).
15. Bohr, H. *Acta Math.* **47,** 237–281 (1926).
16. Kraus, Y. E., Lahini, Y., Ringel, Z., Verbin, M. & Zilberberg, O. *Phys. Rev. Lett.* **109,** 106402 (2012).
17. Verbin, M., Zilberberg, O., Lahini, Y., Kraus, Y. E. & Silberberg, Y. *Phys. Rev. B* **91,** 064201 (2015).
18. Lohse, M. *et al. Nature Phys.* **12,** 350–354 (2015).
19. Nakajima, S. *et al. Nature Phys.* **12,** 296–300 (2016).
20. Lu, H. I. *et al. Phys. Rev. Lett.* **116,** 200402 (2016).
21. Lu, L., Joannopoulos, J. D. & Soljačić, M. *Nature Phys.* **12,** 626–629 (2016).
22. Goldman, N., Budich, J. C. & Zoller, P. *Nature Phys.* **12,** 639–645 (2016).
23. Prodan, E. *Phys. Rev. B* **91,** 245104 (2015).
24. Kraus, Y. E. & Zilberberg, O. *Phys. Rev. Lett.* **109,** 116404 (2012).
25. Dana, I. *Phys. Rev. B* **89,** 205111 (2014).
26. El Hassouani, Y. *et al. Phys. Rev. B* **74,** 035314 (2006).
27. Pang, X. N., Dong, J. W. & Wang, H. Z. *J. Opt. Soc. Am. B* **27,** 2009–2013 (2010).
28. Verbin, M. *et al. Phys. Rev. Lett.* **110,** 076403 (2013).
29. Bellissard, J. *From Number Theory to Physics* 538–630 (Springer, 1992).
30. Levy, E. *et al.* Preprint at http://arxiv.org/abs/1509.04028 (2015).
31. Yang, C. N. *J. Math. Phys.* **19,** 320–328 (1978).
32. Zhang, S.-C. & Hu, J. *Science* **294,** 823–828 (2001).
33. Kraus, Y. E., Ringel, Z. & Zilberberg, O. *Phys. Rev. Lett.* **111,** 226401 (2013).
34. Ben-Abraham, S. I. & Quandt, A. *Phil. Mag.* **91,** 2718–2727 (2011).
35. Bandres, M. A., Rechtsman, M. C. & Segev, M. *Phys. Rev. X* **6,** 011016 (2016).
36. McGrath, R. *et al. J. Phys. Cond. Matter* **22,** 084022 (2010).
37. Tran, D. T. *et al. Phys. Rev. B* **91,** 085125 (2015).
38. Fulga, I., Pikulin, D. & Loring, T. Preprint at http://arxiv.org/abs/1510.06035 (2015).
39. Mourik, V. *et al. Science* **336,** 1003–1007 (2012).
40. Das, A. *et al. Nature Phys.* **8,** 887–895 (2012).
41. Deng, M. *et al. Nano Lett.* **12,** 6414–6419 (2012).
42. Lang, L. J. & Chen, S. *Phys. Rev. B* **86,** 205135 (2012).
43. Tezuka, M. & Kawakami, N. *Phys. Rev. B* **88,** 155428 (2013).
44. Cai, X., Lang, L. J., Chen, S. & Wang, Y. *Phys. Rev. Lett.* **110,** 176403 (2013).
45. DeGottardi, W., Sen, D. & Vishveshwara, S. *Phys. Rev. Lett.* **110,** 146404 (2013).
46. Wang, J. *et al. Phys. Rev. B* **93,** 104504 (2015).
47. Beenakker, C. & Kouwenhoven, L. *Nature Phys.* **12,** 618–621 (2016).

Acknowledgements

We thank H. M. Price for feedback on the article.

Topological states in photonic systems

Ling Lu, John D. Joannopoulos and Marin Soljačić

Optics played a key role in the discovery of geometric phase. It now joins the journey of exploring topological physics, bringing bosonic topological states that equip us with the ability to make perfect photonic devices using imperfect interfaces.

In 1956, a 22-year-old Indian physicist known as Pancharatnam[1] showed that, over the course of a cyclic evolution, the polarization vector of light acquires a phase that is only determined by the area of the cycle on the Poincaré sphere. Three years later, the celebrated Aharonov–Bohm phase was discovered[2]. After more than two decades, Berry[3] generalized these two geometric phases into what we now call the 'Berry phases', which were subsequently demonstrated experimentally in coiled optical fibres[4]. It was immediately realized[5] that the Berry phase is closely connected with the topological interpretation[6] of the integer quantum Hall effect (QHE), which was discovered in 1980. In particular, the signed integers (Z) in the quantized edge conductance are mathematically equivalent to the bulk Chern numbers of the fibre bundle theory in topology (see Commentary by Asorey[7]). Another example of the bulk-edge correspondence was shown by Zak[8], who demonstrated that the bulk Berry phase differentiates the Tamm and Shockley states[9], found in the 1930s, at the ends of one-dimensional (1D) lattices.

In 1988, Haldane proposed a model[10] suggesting that the QHE can be realized in a band structure with time-reversal-symmetry (T) breaking, in which the bands acquire non-zero Chern numbers. This phenomenon is known as the quantum anomalous Hall effect (QAHE). In 2005, Kane and Mele[11] proposed the quantum spin Hall effect (QSHE) by finding that two copies of the Haldane model are also robust, without breaking T. This is possible because the crossing points in the band structures are enforced by the Kramers degeneracy of electrons; that is, there is no modal coupling or anti-crossing. This discovery of the QSHE, which is protected by T, has led to the broad notion of symmetry-protected topological phases. Since then countless new topological phases have been theoretically classified, with a handful being experimentally realized, including 3D topological insulators, topological crystalline insulators and Weyl semimetals. All these topological materials are essentially non-interacting systems of nontrivial band topologies, which are well-described by single-particle band structures. As such, they can also be realized in photonic band dispersions, where photons are the non-interacting bosons.

In this Commentary, we briefly summarize the development of topological photonics[12], discuss its unique characteristics and highlight some of the latest progress in the field.

Photons versus electrons

There is one key distinction between spin-1 bosons and spin-1/2 fermions: the T operator squares to +1 for bosons, whereas it squares to –1 for fermions. As a consequence, the topological classifications with respect to T are different for the two classes of particles. For example, T itself cannot protect a topological phase for bosons[12]. So T-invariant designs always require fine-tuning as discussed later in this Commentary.

Still, there are many advantages to studying band topologies in photonic systems. First, photons have no Fermi levels — the energy scales close to which electronic bands can be studied. In contrast, the whole photonic band structure can be probed using photons of different frequencies. Second, photonic structures can be tuned continuously to create any of the allowed bulk or edge dispersions. The ability to control

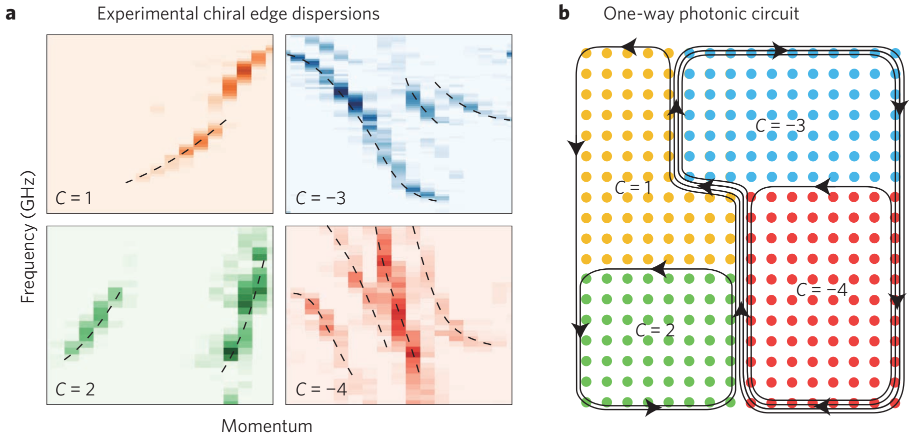

Figure 1 | One-way dispersions and circuit. **a**, Chiral edge dispersions of the one-way waveguides of Chern number (C) 1, 2, –3 and –4, obtained by Fourier-transforming the phase-resolved spatial field profiles of the edge modes. **b**, Illustration of a one-way photonic circuit built by interfacing bulk photonic crystals of different Chern numbers.

these structures also enables the study of disorder physics, quasicrystals (see Commentary by Kraus and Zilberberg[13]) and topological defects — all of which are very difficult to achieve in electronic systems. Third, Maxwell's equations can be solved exactly. Therefore, one can do numerical experiments to support the real experiments. In addition, there is no fundamental length scale in Maxwell's equations: experimentalists can work at any wavelength (optical, terahertz or microwave, for instance) where materials and measurements are most feasible. Lastly, it is easier for photonic research to be translated into applications, as we discuss in the final section of this Commentary.

Another fundamental aspect for electromagnetic fields in the classical regime is the fact that all classical wave fields are intrinsically real-valued (which is the key feature of topological superconductors, as discussed in the Commentary by Beenakker and Kouwenhoven[14]). This ensures identical solutions at opposite frequency and momenta in the wave equations. Such a built-in 'particle–hole' symmetry brings new classes of topological states with respect to the zero frequency, as demonstrated in mechanical systems (see Commentary by Huber[15]) and in magnetoplasmons[16].

One-way waveguides

In 2005, Haldane[17] and Raghu[18] suggested that the band structure of the then-elusive QAHE could be realized in a gyroelectric photonic crystal by gapping a pair of Dirac cones under a static magnetic field. Three years later, Wang *et al.* made a realistic design[19] of a gyromagnetic photonic crystal with a Chern number of 1. The team also performed the experiments[20], using an array of ferrimagnetic rods inside an electromagnet with a field strength of ~0.2 tesla. They made observations at microwave frequencies, which revealed that the edge mode propagates unidirectionally and that the transmission is immune to large metallic scatterers. Skirlo *et al.* recently demonstrated[21] larger Chern numbers in a similar system by gapping multiple pairs of Dirac cones simultaneously. The authors were able to scan not only the amplitude but also the phase of the edge modes and then Fourier-transform them to observe[22] the chiral dispersions of Chern numbers 1, 2, –3 and –4 (Fig. 1a). Such a rich variety of Chern numbers paves the way to integrated one-way photonic circuits, like the one illustrated in Fig. 1b. Unfortunately, because the frequency of gyromagnetic resonance scales with the magnitude of the magnetic field, it is difficult to operate this class of systems towards optical frequencies.

At optical frequencies, T can be broken with time-dependent modulations — also known as a Floquet system. Fang *et al.*[23] proposed an array of resonators whose coupling is controlled by dynamic links. Although such a proposal can in principle be realized by modulating the refractive indices, this is technologically challenging for a large-scale lattice. Rechtsman *et al.* avoided this difficulty[24] by adopting a change of reference frame: from that of the lab to that of the photon. They extended the 2D lattice along the third direction into a waveguide array. The waveguides were spirals, so the photons were modulated as they propagated in the waveguides, thereby

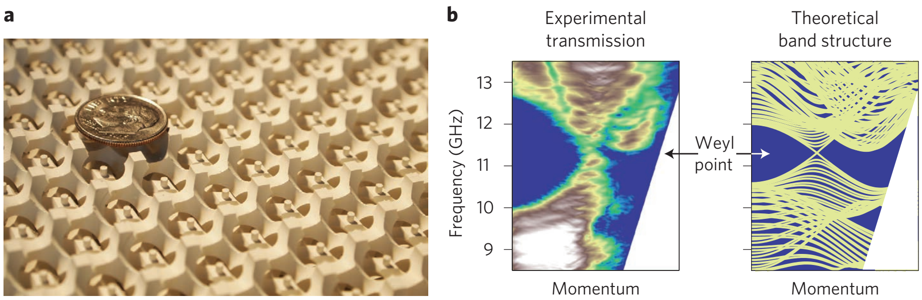

Figure 2 | Weyl points in a gyroid photonic crystal. **a**, The surface of the non-centrosymmetric double-gyroid photonic crystal with a dime on top. **b**, The microwave transmission intensity through the sample, which compares very well to the theoretical projected Weyl band structure.

allowing an effective breaking of T in the photons' reference frame. This experimental demonstration was performed at visible wavelengths.

T-invariant designs

Owing to the difficulty in breaking T at optical frequencies, it is highly desirable to achieve T-invariant 2D topological phases (analogous to the QSHE) with robust counter-propagating helical edge states. However, due to the lack of Kramers degeneracies, two counter-propagating bosonic edge states will generally couple and backscatter. Luckily, photonic systems have a large amount of freedom for designers. Fine-tuning the system parameters could minimize the modal coupling and achieve near-reflectionless propagation in certain designs without breaking T. In the first example, Hafezi *et al.*[25], by assuming the absence of local imperfections, created two copies of the QHE edge states through the use of artificial gauge fields. They connected a 2D array of ring cavities through waveguides with different lengths to mimic the Aharonov–Bohm phase of electrons under a uniform magnetic field, as in the integer QHE. (Liang *et al.*[26] later simplified this system by showing that the waveguides could be identical, as in the QAHE.) The team fabricated[27] this system on a silicon-on-insulator chip using standard CMOS technology and observed the edge states at near-infrared wavelengths. Such a reciprocal system is robust against variations in the resonator frequencies, but it remains vulnerable to local defects[28].

In the second T-invariant 2D system, Khanikaev *et al.*[29] mimicked the QSHE using the two polarization states by matching permittivity (ε) and permeability (μ) values in metamaterials. Such symmetry in the constitutive relations guarantees the separation of polarization modes. Polarized edge states flow in both directions and do not scatter into each other as long as the ratio ε/μ is constant across the whole system. Chen *et al.*[30] realized this proposal by embedding a metamaterial photonic crystal inside a parallel-plate metallic waveguide. Cheng *et al.*[31] showed that certain types of reduction in back-reflection can be achieved through a more practical design that relaxes the strict ε–μ requirement.

In the third example, Wu *et al.*[32] showed theoretically that a topological phase transition can take place in the bulk of a 2D photonic crystal that is invariant under C_6 rotation. Unfortunately, there are no protected edge states for such a crystal because C_6 symmetry cannot be satisfied on a 1D edge. However, by tuning the edge configuration, the anticrossing gap between the two counter-propagating edge states can be very small, thus reducing back-reflection.

Weyl points

Connecting equivalent boundaries of a Brillouin zone in two dimensions gives a toroidal topology. The Chern number of a 2D band can be understood as the number of Berry monopoles inside the torus, which emit quantized Berry flux through the dispersion band. In 3D momentum space, the Berry monopole is a Weyl point — a 3D linear point degeneracy between two bands. Topological charges are robust in the 3D momentum space and can only be annihilated pairwise with the opposite charge: this provides a general way to obtain 3D topological gapped phases. Similar to the edge states of the one-way waveguides, the surface state of a Weyl crystal is also gapless, owing to the non-zero Chern numbers in the bulk.

Lu *et al.*[33] theoretically proposed the existence of Weyl points in a double-gyroid 3D photonic crystal, by breaking either the time or inversion symmetry. In both cases, they obtained the minimal number of Weyl points that are frequency-isolated in the Brillouin zone with topologically nontrivial surface states[34]. Recently, the team experimentally[35] fabricated such a gyroid crystal by breaking inversion symmetry, shown in Fig. 2a. Angle-resolved microwave transmission measurements revealed the linear bulk dispersion (Fig. 2b), which is the signature of the long-sought Weyl quasiparticle originally proposed in 1929.

Weyl points were also predicted in a magnetized plasma by Gao and colleagues[36]. In contrast with regular lattices of discrete translations, this bulk plasma medium is invariant under continuous translations. Recently, Silveirinha[37] showed how Chern numbers could be defined in continuous dispersive media, thus indicating that previously known one-way plasmonic modes have topological origins.

Disorder-immune surface

In 2D band structures, the topological phase transition takes place when the band structure has Dirac cones — gapping these can form bandgaps that support gapless edge states. In three dimensions, such a transition is generally a stable phase of Weyl points. When Weyl points annihilate, 3D topological bandgaps can form, thus supporting gapless surface states.

Lu *et al.*[38] theoretically identified a 3D photonic crystal that supports two Weyl points (of opposite Chern numbers) stabilized at the boundary of the 3D Brillouin zone, and in doing so formed a linear four-fold point degeneracy. Such a 3D Dirac point is protected by the non-symmorphic crystal symmetries of the lattice. The team then broke T to gap the 3D Dirac points while preserving the glide reflection, which is a non-symmorphic operation that can be preserved on the surface. The resulting magnetic crystal supported a single Dirac cone on its surface protected by the glide reflection — this was the first material prediction of a 3D gapped bosonic topological phase, characterized by a Z_2 (binary) topological invariant. It can be regarded as the bosonic analogue of either 3D topological insulators featuring single-Dirac-cone surfaces or topological crystalline insulators protected by crystal symmetries.

A single-Dirac-cone surface state remains gapless under random disorder of arbitrary kind. There always exists an extended state delocalized on the magnetic surface, such as that illustrated in Fig. 3b. Without disorder, the surface photon propagates as a massless Dirac particle in two dimensions. In a disordered region, the Dirac particle acquires a mass if the glide reflection is locally broken. Depending on the nature of the disorder, this mass term can be either positive or negative. The random distribution of disorder ensures that the masses of opposite signs are equally populated on the surface. At the 1D interface, where the mass changes sign, a massless solution must exist. It turns out that this solution is identical to that for chiral edge states in a one-way waveguide. Consequently, these one-way edges states percolate the whole surface, leading to the absence of Anderson localization.

Potential applications

The quality of traditional optical elements relies on the ability to polish optically smooth surfaces. In nanophotonics, processes such as lithography, deposition and etching define the quality of the devices. In both cases, fabrication capability plays a critical role in achieving state-of-the-art device performance. Today, topology opens up a whole new degree of freedom. The transport of many topological interfacial states is immune to fabrication imperfections; this has been shown in the above examples of one-way waveguides on the 1D edge and of single-Dirac-cone states on the 2D surface. With one-way waveguides, we need not worry about waveguide bends, coupling efficiencies or the integration of isolators. The 2D topological surface could potentially be used in robust surface sensing and florescence counting. In contrast with a regular optical system, in which the number of states grows rapidly with the system size, the number of modes at Dirac and Weyl points does not scale with the cavity size; this unique feature (among other benefits) might enable single-mode lasers over a larger area or volume. Furthermore, the topological states absent of Anderson localizations may open new opportunities for slow-light[39] technology, which has so far been hindered by the localization of light due to disorder.

The search for topological interfacial states has also inspired the re-examination of other robust phenomena in optics. For example, Zhen *et al.*[40] pointed out that the robust optical bound states in the continuum are, in fact, the vortex cores of the far-field polarization, in which the topological invariant is the winding number of the polarization vector. These topological bound states, which are quite different from the other topological phenomena described in this Commentary, can be used to generate vector beams whose beam angles can be continuously steered.

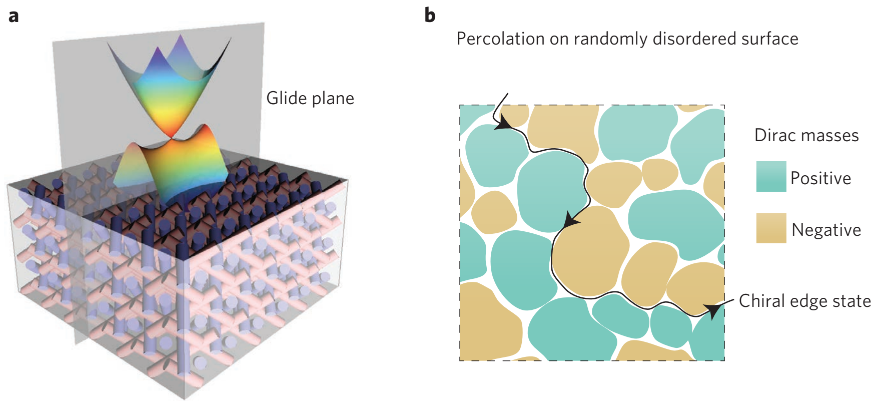

Figure 3 | A single-Dirac-cone surface state immune to random disorder. **a**, Illustration of a 3D magnetic photonic crystal supporting a single Dirac cone on its surface, protected by a glide-reflection plane. **b**, The surface state remains delocalized under any disorder that, on average, does not break the glide-reflection symmetry.

Future directions

In two dimensions, there is an urgent need to achieve one-way waveguides at optical wavelengths. In three dimensions, Weyl points and the glide-protected single-Dirac-cone surface states indicate the various 3D topological phases that could be protected by spatial symmetries. There are 230 space groups and 1,651 magnetic groups. By controlling the photon lifetime, one could explore the opportunities in non-Hermitian systems[41]. In addition, our experience with photonic topological states can be readily applied to other bosonic particles, such as phonons[15], magnons, excitons, polaritons, plasmons[16,36] and cold atoms (see Progress Article by Goldman *et al.*[42]). Last but not least, interactive topological phases of light could be achieved by using highly nonlinear elements to mediate photon interactions, thereby shedding light on many-body physics such as the fractional QHE[43]. ❐

Ling Lu is at the Institute of Physics, Chinese Academy of Sciences/Beijing National Laboratory for Condensed Matter Physics, Beijing 100190, China. John D. Joannopoulos and Marin Soljačić are in the Department of Physics, Massachusetts Institute of Technology, Cambridge 02139, USA. e-mail: linglu@iphy.ac.cn

References

1. Pancharatnam, S. *Proc. Indiana Acad. Sci. A* **44,** 247–262 (1956).
2. Aharonov, Y. & Bohm, D. *Phys. Rev.* **115,** 485–491 (1959).
3. Berry, M. V. *Proc. R. Soc. A* **392,** 45–57 (1984).
4. Tomita, A. & Chiao, R. Y. *Phys. Rev. Lett.* **57,** 937–940 (1986).
5. Simon, B. *Phys. Rev. Lett.* **51,** 2167–2170 (1983).
6. Thouless, D. J., Kohmoto, M., Nightingale, M. P. & den Nijs, M. *Phys. Rev. Lett.* **49,** 405–408 (1982).
7. Asorey, M. *Nature Phys.* **12,** 616–618 (2016).
8. Zak, J. *Phys. Rev. Lett.* **62,** 2747–2750 (1989).
9. Shockley, W. *Phys. Rev.* **56,** 317–323 (1939).
10. Haldane, F. D. M. *Phys. Rev. Lett.* **61,** 2015–2018 (1988).
11. Kane, C. L. & Mele, E. J. *Phys. Rev. Lett.* **95,** 226801 (2005).
12. Lu, L., Joannopoulos, J. D. & Soljačić, M. *Nature Photon.* **8,** 821–829 (2014).
13. Kraus, Y. E. & Zilberberg, O. *Nature Phys.* **12,** 624–626 (2016).
14. Beenakker, C. & Kouwenhoven, L. *Nature Phys.* **12,** 618–621 (2016).
15. Huber, S. D. *Nature Phys.* **12,** 621–623 (2016).
16. Jin, D. *et al.* Preprint at http://arxiv.org/abs/1602.00553 (2016).
17. Haldane, F. D. M. & Raghu, S. *Phys. Rev. Lett.* **100,** 013904 (2008).
18. Raghu, S. & Haldane, F. D. M. *Phys. Rev. A* **78,** 033834 (2008).
19. Wang, Z., Chong, Y. D., Joannopoulos, J. D. & Soljačić, M. *Phys. Rev. Lett.* **100,** 013905 (2008).
20. Wang, Z., Chong, Y. D., Joannopoulos, J. D. & Soljačić, M. *Nature* **461,** 772–775 (2009).
21. Skirlo, S., Lu, L. & Soljačić, M. *Phys. Rev. Lett.* **113,** 113904 (2014).
22. Skirlo, S. A. *et al. Phys. Rev. Lett.* **115,** 253901 (2015).
23. Fang, K., Yu, Z. & Fan, S. *Nature Photon.* **6,** 782–787 (2012).
24. Rechtsman, M. C. *et al. Nature* **496,** 196–200 (2013).
25. Hafezi, M., Demler, E. A., Lukin, M. D. & Taylor, J. M. *Nature Phys.* **7,** 907–912 (2011).
26. Liang, G. Q. & Chong, Y. D. *Phys. Rev. Lett.* **110,** 203904 (2013).
27. Hafezi, M., Mittal, S., Fan, J., Migdall, A. & Taylor, J. M. *Nature Photon.* **7,** 1001–1005 (2013).
28. Gao, F. *et al. Nature Commun.* **7,** 11619 (2016).
29. Khanikaev, A. B. *et al. Nature Mater.* **12,** 233–239 (2013).
30. Chen, W. *et al. Nature Commun.* **5,** 5782 (2014).
31. Cheng, X. *Nature Mater.* **15,** 542–548 (2016).
32. Wu, L. H. & Hu, X. *Phys. Rev. Lett.* **114,** 223901 (2015).
33. Lu, L., Fu, L., Joannopoulos, J. D. & Soljačić, M. *Nature Photon.* **7,** 294–299 (2013).
34. Fang, C., Lu, L., Liu, J. & Fu, L. *Nature Phys.* http://dx.doi.org/10.1038/nphys3782 (2016).
35. Lu, L. *et al. Science* **349,** 622–624 (2015).
36. Gao, W. *et al.* Preprint at http://arxiv.org/abs/1511.04875 (2015).
37. Silveirinha, M. G. *Phys. Rev. B* **92,** 125153 (2015).
38. Lu, L. *et al. Nature Phys.* **12,** 337–340 (2016).
39. Yang, Y. *et al. Appl. Phys. Lett.* **102,** 231113 (2013).
40. Zhen, B., Hsu, C. W., Lu, L., Stone, A. D. & Soljačić, M. *Phys. Rev. Lett.* **113,** 257401 (2014).
41. Zeuner, J. M. *et al. Phys. Rev. Lett.* **115,** 040402 (2015).
42. Goldman, N., Budich, J. C. & Zoller, P. *Nature Phys.* **12,** 639–645 (2016).
43. Umucalılar, R. O. & Carusotto, I. *Phys. Rev. Lett.* **108,** 206809 (2012).

Acknowledgements

We thank P. Rebusco for critical reading and editing of the manuscript. J.D.J. was supported in part by the US Army Research Office through the Institute for Soldier Nanotechnologies under contract W911NF-13-D-0001. L.L. was supported in part by the Materials Research Science and Engineering Center Program of the NSF under award DMR-1419807. M.S. and L.L. (analysis and reading of the manuscript) were supported in part by the MIT Solid-State Solar-Thermal Energy Conversion Center and Energy Frontier Research Center of DOE under grant DE-SC0001299. L.L. is also supported by the National Thousand-Young-Talents Program of China.