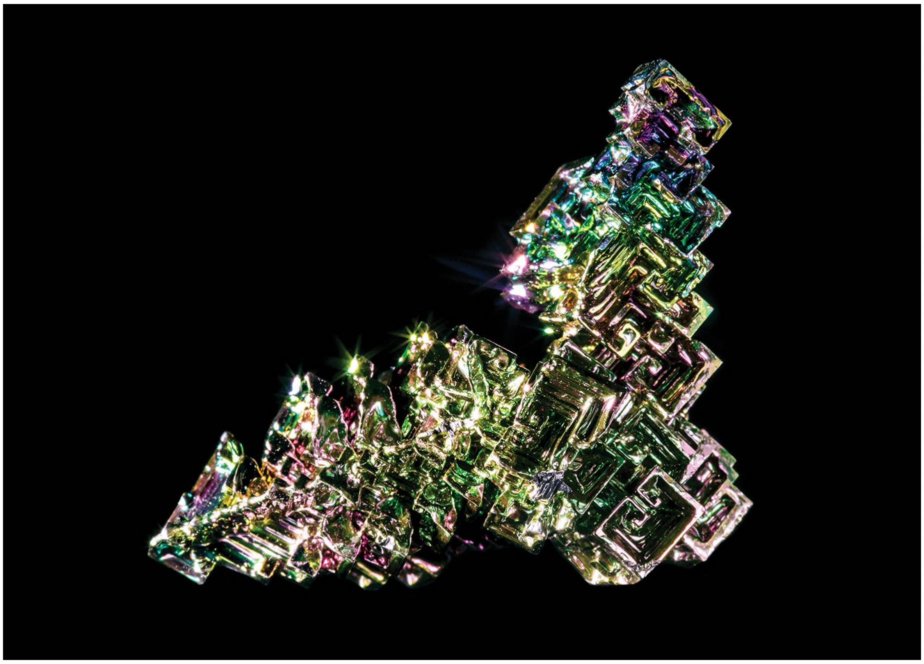


Fig. 1 | Bismuth crystal. Petr Jan Juracka/Science Photo Library.

picture. Now, many topological materials can be found beyond the original paradigm. In this sense, a simple band inversion triggering the existence of a Shockley-type surface state can also be classified as a change in topology accompanied by a change in the respective topological invariant[6].

In addition, a three-dimensional cube has not only two-dimensional surfaces, but also one-dimensional edges (hinges) and zero-dimensional corners. The concept of topological surface states can also be extended to lower dimensions with respect to the bulk. This is what higher-order topology is all about. It describes the influence of a three-dimensional bulk on the one-dimensional edges (hinges) or even zero-dimensional corners. For bismuth, it means that although the band structure is topologically trivial from a first-order perspective, the rotation and inversion symmetries together protect its higher-order topology, eventually giving rise to a one-dimensional gapless state on its edge. This finding expands the list of elusive properties of bismuth and continues the classification of topological phases of matter[4].

Applying the concept of topology in physics has proven so powerful that David Thouless, Duncan Haldane and Michael Kosterlitz were awarded the 2016 Nobel Prize in Physics 'for theoretical discoveries of topological phase transitions and topological phases of matter'. Their work dates back decades before the more recent surge in interest. What is amazing about topology in physics is not so much that it works — it kind of has to work — but what we can learn from it. It provides a path for new discoveries and predictive understanding.

Some secrets about bismuth have now been revealed, but they were by no means obvious. It is radioactive with a half-life of more than 10^{19} years[2], which exceeds the age of the Universe by about ten orders of magnitude. It becomes superconducting at 0.53 mK at ambient pressure[3]. In addition, few people probably know about its technological relevance in phase-change materials and in thermoelectrics. And now it is a higher-order topological material. It will be exciting to see what comes next and what role bismuth will play in this. We just have to keep going. The closer we look, the bigger the picture. ❒

Christian R. Ast
Max-Planck-Institut für Festkörperforschung, Stuttgart, Germany.
e-mail: c.ast@fkf.mpg.de

Published online: 30 July 2018
https://doi.org/10.1038/s41567-018-0258-x

References

1. Édel'man, V. S. *Sov. Phys. Uspekhi* **20**, 819–835 (1977).
2. de Marcillac, P., Coron, N., Dambier, G., Leblanc, J. & Moalic, J.-P. *Nature* **422**, 876–878 (2003).
3. Prakash, O., Kumar, A., Thamizhavel, A. & Ramakrishnan, S. *Science* **355**, 52–55 (2016).
4. Schindler, F. et al. *Nat. Phys.* https://doi.org/10.1038/s41567-018-0224-7 (2018).
5. Hsieh, D. et al. *Nature* **452**, 970–974 (2008).
6. Yan, B. et al. *Nat. Commun.* **6**, 10167 (2015).

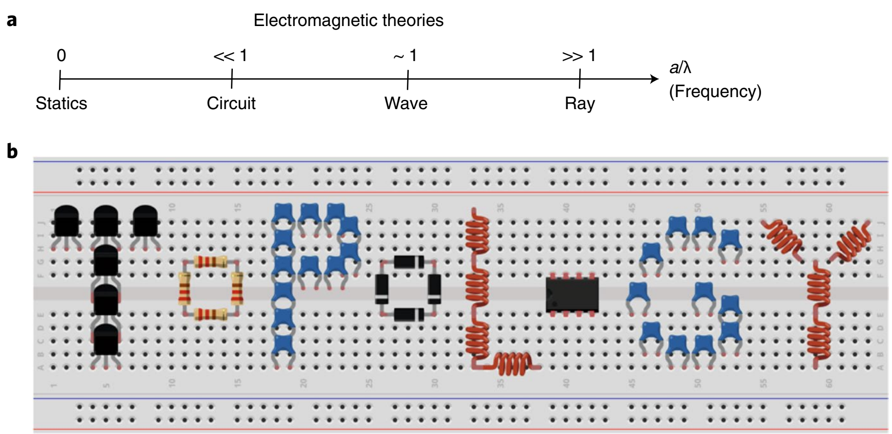
PHOTONICS

Topology on a breadboard

The realization of a new topological state using an electrical-circuit approach establishes a flexible scheme that should enable further explorations into uncharted territory and, equally importantly, make experiments with topological states more broadly accessible.

Ling Lu

The Haldane model[1], introduced by 2016 Nobel laureate Duncan Haldane, epitomizes the importance of band topology, that is, the quantized global configuration of wavefunctions on dispersion bands. Although this model is known as the theory of the quantum anomalous Hall effect, its topological properties — non-trivial bulk bands and one-way edge states — do not require quantum mechanisms and can exist in any system with broken time-reversal symmetry. In this sense, so-called Chern insulators include all systems with a gapped bulk dispersion relation of a non-zero Chern number. With this in mind, Haldane translated his model to magnetic photonic crystals[2], to stimulate



c

Element	Picture	Symbol	Function	
Inductor (L)			Resonance	Hermitian
Capacitor (C)				
Resistor (R)			Loss	Non-Hermitian
Amplifier (U)			Gain	
Diode (D)			Non-reciprocity	
Transistor (T)			Non-linearity	
Superconducting qubit (Q)			Quantum circuit	

Fig. 1 | Exploring topological states with electrical circuits. a, Electromagnetic theories used in different spectral regions according to the ratio between the characteristic structural length *a* and the working wavelength *λ*. **b**, Conceptual sketch of topological experiments on a breadboard. Credit: Hengbin Cheng, created using fritzing.org software. **c**, Components that can be integrated in topological circuitry.

its first implementation. Indeed, the first experiment, on a microwave-based platform[3], came four years before its condensed-matter counterpart[4]. In both cases there exists an edge channel for photons or electrons to propagate magically with zero scattering loss. The difference in experimental complexity can be understood by comparing their working wavelengths, which is in the centimetre range for microwaves and a few atomic layers for molecular beam epitaxy. Reporting in *Nature Physics*, Stefan Imhof and collaborators[5] have expanded the photon wavelength to radiofrequencies (therefore to hundreds of metres), where the Maxwell equations greatly simplify into circuit theories as Kirchhoff's rules or transmission-line analysis. This progress[5–9] in topological photonics[10] creates unique possibilities for richer physics, innovative engineering and science outreach.

Maxwell's equations are the unified description across the entire electromagnetic spectrum, but for most real-life applications solving them directly is either not necessary or not computationally affordable. As illustrated in Fig. 1a, when the characteristic length *a* of the problem is much larger than the working wavelength *λ*, then geometrical optics or ray tracing are used — for example, in lens designs and in rendering motion pictures with special effects. On the other extreme, $a/\lambda \ll 1$, circuit laws work perfectly well in engineering our electronic devices and computer chips. Compared with the intermediate spectral regime, $a/\lambda \sim 1$, where the full-wave solutions are necessary to study problems such as photonic crystals, circuit technology is much more mature and can rely on existing infrastructures and commercial components.

Imhof et al.[5] showcased the power of the circuit approach by experimentally realizing the newly proposed quadrupole topological state[11], which involves four corner modes that are protected by spatial symmetries when both the two-dimensional bulk and the one-dimensional edges are simultaneously gapped. Such bulk–corner correspondence reduces one more spatial dimension than the usual bulk–edge correspondence. The quadrupole corner modes have therefore attracted substantial attention, as the first example of high-order topological 'insulators'. The original theory[11] was initially written as a tight-binding model. However, localized orbits are usually not good bases for photons, unless a series of coupled uniform high-quality resonators can be constructed, which is not simple. However, on a printed circuit board, the task is much more manageable. Wiring up pairs of inductors (L) and capacitors (C) of the same specifications forms a network of LC resonators, which naturally corresponds to a tight-binding model. Furthermore, the key ingredient of the quadrupole state requires a π-flux inside each unit cell, which is a tall order on the atomic scale. In contrast, it is straightforward to achieve the unit-cell flux on the circuit board, operating at 2.8 MHz, by adjusting the coupling wires on each side of the LC resonators. Similar constructions were also adopted by two other groups to demonstrate the same higher-order topological physics: one at 1.4 GHz (ref. [12]), the other at 74 kHz in coupled mechanical resonators[13].

Looking ahead, topological circuits could facilitate many truly exciting experiments currently unattainable in existing platforms by simply connecting a few more electric wires between the resonators on the circuit board. For example, lattices in four or five dimensions can be readily realized. Non-trivial manifolds — including Möbius strips, tori, Klein bottles and real projective planes — are also within immediate reach. Controlled long-ranged couplings in lattice models can now be naturally included. Moreover, there are many more ingredients than the LC resonances that can be integrated (Fig. 1c): resistors and amplifiers can be used to study non-Hermitian phenomena, diodes provide a mechanism to break reciprocity, and transistors are highly nonlinear. In addition, the use of superconducting qubits on a microwave chip is one of the most promising routes to quantum computing. Working at the same frequency and on the same platform, topological photonics could potentially benefit, or benefit from, circuit quantum electrodynamics[6].

Last but not least, experimental schemes using off-the-shelf components and toolkits can stimulate broader interest in this Nobel-winning research topic, including from the public, such as non-science majors in a hobby shop. For instance, intriguing lab classes could be created for high-school students to study topological phenomena using lumped elements on a breadboard. ❐

Ling Lu
Institute of Physics, Chinese Academy of Sciences/ Beijing National Laboratory for Condensed Matter Physics, Beijing, China.
e-mail: linglu@iphy.ac.cn

Published online: 4 September 2018
https://doi.org/10.1038/s41567-018-0235-4

References
1. Haldane, F. D. M. *Phys. Rev. Lett.* **61**, 2015–2018 (1988).
2. Haldane, F. D. M. & Raghu, S. *Phys. Rev. Lett.* **100**, 013904 (2008).
3. Wang, Z., Chong, Y. D., Joannopoulos, J. D. & Soljačić, M. *Nature* **461**, 772–775 (2009).
4. Chang, C.-Z. et al. *Science* **340**, 167–170 (2013).
5. Imhof, S. et al. *Nat. Phys.* https://doi.org/10.1038/s41567-018-0246-1 (2018).
6. Roushan, P. et al. *Nature* **515**, 241–244 (2014).
7. Ningyuan, J., Owens, C., Sommer, A., Schuster, D. & Simon, J. *Phys. Rev. X* **5**, 021031 (2015).
8. Albert, V. V., Glazman, L. I. & Jiang, L. *Phys. Rev. Lett.* **114**, 173902 (2015).
9. Lee, C. H. et al. *Commun. Phys.* https://doi.org/10.1038/s42005-018-0035-2 (2018).
10. Lu, L., Joannopoulos, J. D. & Soljačić, M. *Nat. Photon.* **8**, 821–829 (2014).
11. Benalcazar, W. A., Bernevig, B. A. & Hughes, T. L. *Science* **357**, 61–66 (2017).
12. Peterson, C. W., Benalcazar, W. A., Hughes, T. L. & Bahl, G. *Nature* **555**, 346–350 (2018).
13. Serra-Garcia, M. et al. *Nature* **555**, 342–345 (2018).

CONDENSED-MATTER PHYSICS

Super spins

Applications of spintronics often require angular momentum to be moved from place to place. A possible observation of spin superfluidity may point the way toward the transport of spin angular momentum across an insulating sample with no dissipation or energy loss.

Joshua Folk

The remarkable phenomena of superfluidity and superconductivity — levitating magnets, enormous currents flowing through thin wires with no Joule heating, liquid helium flowing with zero viscosity through the narrowest of openings or creeping mysteriously over the walls of an open bucket — continue to spark the imaginations of all who encounter them, long after their discoveries in the first half of the twentieth century. Writing in *Nature Physics*, Petr Stepanov and co-workers[1] report the possible realization of a related 'super' phenomenon, in which spin currents may be transported without dissipation across a flake of graphene. Spin superfluidity involves the flow not of electrons or Cooper pairs (as in superconductivity), or of atoms (as in superfluidity), but of pure angular momentum, without an accompanying flow of particles.

Spin superfluidity has been predicted to emerge in charge-neutral graphene when high magnetic fields give rise to an insulating state with massive degeneracy, thereby stabilizing a correlated spin texture known as a canted antiferromagnet. In this state, electrons on alternate carbon atoms in the hexagonal lattice order antiferromagnetically in the plane, spin left, spin right, spin left, and so on, with a slight cant out of the plane due to the external magnetic field. Ignoring sample defects and boundaries, the antiferromagnetic coupling in graphene should be isotropic, so the direction of the anti-alignment of the spins — the Néel vector — could lie anywhere in the graphene plane. As long as the sample is in equilibrium, this vector should point in the same direction throughout the lattice, locked by interatomic antiferromagnetic couplings.

The angle of the local Néel vector is associated with an order parameter for the spin superfluid state, analogous to the phase of the complex order parameters associated with superconductivity or superfluidity. The dissipationless flows of charge currents in a superconductor or atoms in a superfluid are associated with gradients of that phase across the sample. In a spin superfluid, the Néel vector begins to precess when angular momentum is injected into one area. If angular momentum is simultaneously removed from another area, there will be a relative twist in the Néel vector angle across the sample associated with the angular momentum flow from injection to removal points. Thus, the mathematical description of angular momentum flow in spin superfluidity, due to a gradient in the Néel vector angle, closely resembles that of charge current flow in a superconductor due to a gradient in the phase of the order parameter.

Stepanov and co-workers aim to drive a flow of angular momentum through graphene (Fig. 1) by applying a voltage between spin-up and spin-down edge states adjacent to a canted antiferromagnetic region of their device. In order for carriers in the higher-potential edge state to relax down to the lower-potential edge state, they must flip their spin, transferring their angular momentum to the lattice or, ideally, to the canted antiferromagnetic state. The detection scheme is based on an inverse of this effect, measuring a voltage difference between spin-polarized edges on the other side of the central region that results from a flow of angular momentum into the edge states.

Applying a voltage across separately contacted spin-polarized edge states is a proven technique for supplying angular momentum to the lattice in conventional semiconductors, enabling, for example, electrically driven nuclear polarization in GaAs (ref. [2]). Unfortunately for Stepanov and co-workers, the coupling between spin-polarized edge states in graphene is very weak, so a large drive voltage was required to generate a detectable signal. In this situation, one must always worry that thermal or other higher-order effects may come into play.

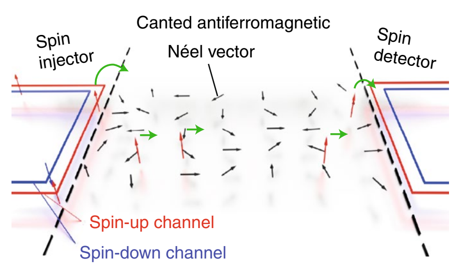


Fig. 1 | Illustration of the Stepanov device. On the left side of the image, electrons tunnel from the spin-up to the spin-down channel (or vice versa, depending on bias), injecting their angular momentum into the central canted antiferromagnetic region and causing the local Néel vector to rotate. On the right, angular momentum can exit the central region, giving rise to a bias differential between the spin-up and spin-down channels on that side. Credit: Adapted from ref. [1], Macmillan Publishers Ltd

An additional challenge in the detection of spin superfluidity is that angular momentum within the electronic system is not a conserved quantity. Even a dissipationless flow of angular momentum across a sample will decay on a characteristic length scale similar to the spin relaxation length for